Graph $f(x)=\dfrac{x+1}{x^2}$ using the process shown in lecture and in the website handout. **SCORE: _____ / 45 PTS**

The first and second derivatives are $f'(x)=-x^{-2}-2x^{-3}$ and $f''(x)=2x^{-3}+6x^{-4}$.

<u>Complete the table below, after showing relevant work (except for entries marked ★).</u>
<u>You will NOT receive credit for the entries in the table if the relevant work is missing.</u>

★ Domain	★ Discontinuities	$x-$ and $y-$ intercepts	One sided limits at each discontinuity (write using proper limit notation)	
$x \neq 0$	$x=0$	X-INT : $x=-1$ Y-INT : NONE	$\lim_{x\to 0^+} \frac{x+1}{x^2} = \lim_{x\to 0^-} \frac{x+1}{x^2} = \infty$	
Horizontal Asymptotes	Intervals of Increase	Intervals of Decrease	Intervals of Upward Concavity	Intervals of Downward Concavity
$y=0$	$(-2,0)$	$(-\infty,-2),(0,\infty)$	$(-2,0),(0,\infty)$	$(-\infty,-3)$
Vertical Tangent Lines (x – value only)	Horizontal Tangent Lines (x – value only)	Local Maxima (x – value only)	Local Minima (x – value only)	Inflection Points (x – value only)
NONE	$x=-2$	NONE	$x=-2$	$x=-3$

X-INT: $\frac{x+1}{x^2}=0 \longrightarrow x+1=0 \longrightarrow x=-1$

Y-INT : $f(0)$ DNE

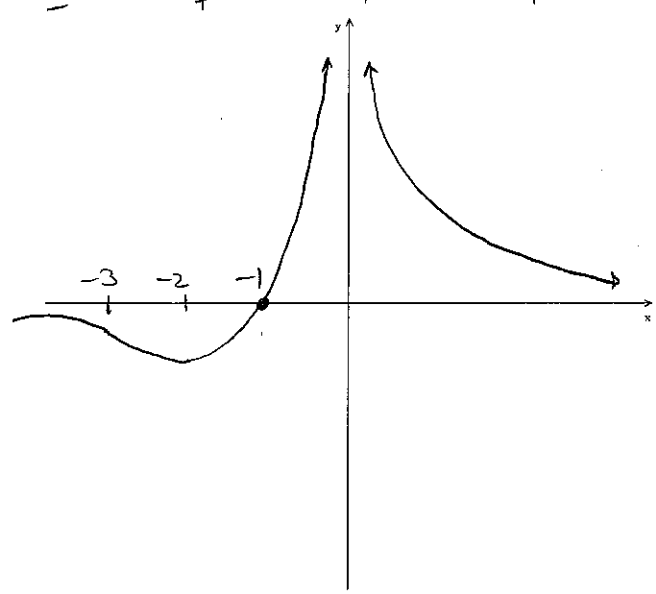
$\lim_{x\to 0^\pm} \frac{x+1}{x^2} = \infty \quad \left(\frac{1}{0^+}\right)$

$\lim_{x\to\pm\infty}\left(\frac{1}{x}+\frac{1}{x^2}\right)=0+0=0$

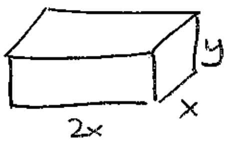
$f'(x)=-x^{-3}(x+2)$ DNE @ $x=0 \notin$ DOMAIN
$=0$ @ $x=-2$

$f''(x)=2x^{-4}(x+3)$ DNE @ $x=0 \notin$ DOMAIN
$=0$ @ $x=-3$

f'	−	−	+	−
f''	−	+	+	+
	-3	-2	0	
$-x^{-3}$	+	+	+	−
$x+2$	−	−	+	+
$2x^{-4}$	+	+	+	+
$x+3$	−	+	+	+



A rectangular storage container with an open top is to have a volume of 60 cubic feet. SCORE: _____ / 30 PTS
The length of the base is twice the width. Material for the base costs $\$10$ per cubic foot. Material for the sides costs $\$6$ per cubic foot.
Find the cost of materials for the cheapest such container.



MINIMIZE COST OF MATERIALS $= C = 20x^2 + 36xy$

$2x^2y = 60$

$y = \frac{30}{x^2}$

$C = 20x^2 + \frac{1080}{x}$

$x \in (0, \infty)$

$C' = 40x - \frac{1080}{x^2} = 0$

$\frac{40}{x^2}(x^3 - 27) = 0$

$x = 3$

WHEN $x = 3$, $C = 20(9) + \frac{1080}{3} = 540$

$\lim\limits_{x \to 0^+} \left(20x^2 + \frac{1080}{x}\right) = \infty$

$\lim\limits_{x \to \infty} \left(20x^2 + \frac{1080}{x}\right) = \infty$

THE CHEAPEST CONTAINER REQUIRES \$540 OF MATERIALS

Prove that the equation $2x = \cos x$ has exactly one solution. SCORE: _____ / 15 PTS

LET $f(x) = 2x - \cos x$, WHICH IS CONT. + DIFF. SINCE IT IS THE DIFFERENCE OF CONT. + DIFF. POLYNOMIAL + TRIG FUNCTIONS

$f(0) = -1$, $f(\pi) = 2\pi + 1$

$f(0) < 0 < f(\pi)$, SO BY IVT, $f(c) = 0$ FOR SOME $c \in (0, \pi)$

SUPPOSE $f(x) = 0$ FOR AT LEAST 2 VALUES $a < b$.

SO $f(a) = f(b) = 0$

BY ROLLE'S THEOREM, $f'(d) = 0$ FOR SOME $d \in (a, b)$

BUT $f'(d) = 2 + \sin d$ AND $1 \le 2 + \sin d \le 3$ (CONTRADICTION)

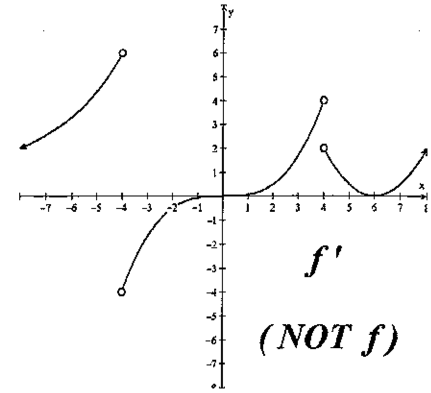
SO $f(x) = 0$ FOR ONLY 1 VALUE OF x

IE. $2x - \cos x = 0$ OR $2x = \cos x$ HAS EXACTLY 1 SOLUTION

$f(x)$ is a continuous function whose derivative $f'(x)$ is shown on the right.

SCORE: _____ / 18 PTS

The following questions are about the function f, <u>**NOT THE FUNCTION f'**</u>.



f'

(NOT f)

[a] Find the intervals over which f is concave down.
<u>**Justify your answer very briefly WITHOUT referring to f''.**</u>

f' DECREASING ON $(4, 6)$

[b] Find the $x-$coordinates of all local extrema of f, and determine whether each is a local maximum or a local minimum.
<u>**Justify your answer very briefly.**</u>

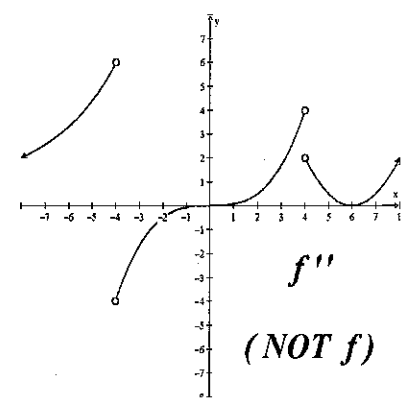
$f' = 0$ AND CHANGES FROM $-$ TO $+$ AT $x = 0 \rightarrow$ LOCAL MIN

f' DNE AND CHANGES FROM $+$ TO $-$ AT $x = -4 \rightarrow$ LOCAL MAX

$f(x)$ is a continuous and differentiable function whose second derivative $f''(x)$ is shown on the right.

SCORE: _____ / 12 PTS

The following questions are about the function f, <u>**NOT THE FUNCTION f''**</u>.



f''

(NOT f)

[a] Find the $x-$coordinates of all inflection points of f. <u>**Justify your answer very briefly.**</u>

f'' CHANGES FROM $+$ TO $-$ AT $x = -4$

$-$ $+$ $x = 0$

[b] If $f'(-5) = 0$,
what does the Second Derivative Test tell you about the point $(-5, f(-5))$?
<u>**Justify your answer very briefly.**</u>

$f''(-5) > 0 \rightarrow (-5, f(-5))$ IS LOCAL MIN

Evaluate $\lim_{x\to 0}\frac{1+x\cos x-e^x}{1-\cos 2x}$. $\frac{1+0-1}{1-1} \to \frac{0}{0}$

SCORE: _____ / 15 PTS

Your answer should be a number, ∞, −∞ or DNE (only if the first three answers do not apply).

$$\lim_{x\to 0}\frac{\cos x - x\sin x - e^x}{2\sin 2x} \qquad \frac{1-0-1}{0} \to \frac{0}{0}$$

$$\lim_{x\to 0}\frac{-\sin x - \sin x - x\cos x - e^x}{4\cos 2x}$$

$$= \frac{0-0-0-1}{4}$$

$$= -\frac{1}{4}$$

THE LIMIT IS $-\frac{1}{4}$

You wish to use Newton's Method to solve the equation $x^2 - 3 = 4x$.

SCORE: _____ / 15 PTS

[a] If you use the initial approximation $x_0 = 1$, find the values of x_1 and x_2.

$x^2-4x-3=0 \quad f(x)=x^2-4x-3$

$f'(x)=2x-4$

$$x_1 = x_0 - \frac{x_0^2-4x-3}{2x_0-4} = 1-\frac{-6}{-2} = -2$$

$$x_2 = -2-\frac{9}{-8} = -\frac{7}{8}$$

[b] What value(s) of x_0 will cause Newton's Method to fail immediately ? **Justify your answer very briefly.**

$2x-4=0 \to x=2$